Multiuser Beamforming Optimization via Maximizing Modified SLNR with Quantized CSI Feedback

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***Abstract*—In this paper, we investigate the beamforming optimization for the downlink of multiuser MISO (MU-MISO) system with finite-rate feedback, where the transmitter is only aware of a delayed and quantized version of the channel state information (CSI). By taking into consideration both CSI quantization error and feedback delay, we propose an improved beamforming optimization criterion by maximizing the signal-to-leakage-and-noise ratio (SLNR) averaged over CSI imperfection. A closed-form expression for the optimized beamforming design is then presented. Numerical results verify that the proposed scheme is more robust to the CSI imperfection in finite-rate feedback system than conventional SLNR-based beamforming scheme as well as another well-known zero-forcing beamforming (ZFBF) scheme in terms of both sum rate and bit error rate (BER).**

I. Introduction

How to improve the system capacity and transmission reliability is one of the main issues that the 4th Generation Mobile Communication (4G) will have to tackle. Benefitting from its potential to utilize the space resource to the full, multiple-input multiple-output (MIMO) technique owns bright prospects in the future wireless communications and actually is under more and more investigations now. Assuming the base station (BS) has M transmit antennas to simultaneously serve K users each with only one receive antenna due to the limitation of cost and product size, it is known that the multiplexing gain of this MISO system could reach $min\{M, K\}$ times over that of the single-input single-output (SISO) scenario. Dirty paper coding (DPC), a multiuser encoding strategy based on interference presubstraction proposed in [1], is one way to achieve this limit. However, DPC is hard to be implemented in practical systems due to its high computational burden of successive encodings and decodings.

Recently, beamforming has been acknowledged as a promising strategy for achieving full multiplexing gain with significantly reduced computational complexity relative to DPC. In [2], it reveals that if the beamforming vectors are selected optimally, the MISO system capacity approaches to that of DPC when the number of users K goes to infinity. However, finding the optimal beamforming vectors through maximizing the signal-to-interference-and-noise ratio (SINR) is still a nonconvex optimization problem [3].

In order to further reduce the computational complexity, a kind of linear beamforming scheme is found by inverting the composite channel matrix of the system, i.e. zero-forcing beamforming (ZFBF), and performs quite well at high signal-to-noise ratios (SNRs). However, when applying ZFBF in a practical limited feedback system, the system achievable rate degrades due to imperfect CSI at the transmitter (CSIT) and becomes upper-bounded when the number of feedback bits is fixed with growing SNR to infinity [4]. Similar phenomenon can be found with another linear beamforming scheme called minimum mean square error beamforming (MMSEBF) in [5]. Fortunately, several literature like [6]–[8] have put forward robust MMSEBF schemes to combat the CSI imperfection.

Different from ZFBF scheme intending to increase the SINR through eliminating the co-channel interference (CCI), a new criterion of SLNR maximization was proposed in [9] and [10] for beamforming design. Since SLNR takes into account both CCI and background noise for individual user beamforming optimization, it generally outperforms ZFBF scheme. However, the cost of SLNR is that it requires full CSI of users including both the channel direction information (CDI, or normalized CSI) and the channel magnitude. Due to this reason, most existing works still apply ZFBF instead of SLNR in finite-rate feedback system with only quantized CDI available at the transmitter.

The main contribution of this paper is to design beamforming vectors based on maximizing SLNR in finite-rate feedback system where only a delayed and quantized version of the CDI is available at the BS. We propose a modified SLNR-based multiuser beamforming optimization by exploiting the statistic characteristics of CSI feedbacks. We derive a closed-form beamforming design for the finite-rate feedback system via maximizing the newly modified SLNR metric and utilizing *Jensen's inequality* [11]. Numerical results also show that the newly proposed scheme is *more robust to the quantization error and feedback delay in finite-rate feedback systems* than the previous SLNR-based scheme in [9] and [10] as well as the most popular ZFBF technique.

Notations: In this paper, we use $\mathbf{x}^{\mathrm{T}}$, $\mathbf{x}^{\dagger}$ and $\|\mathbf{x}\|$ to denote the transpose, the conjugate transpose and the Euclidean norm of vector $\mathbf{x}$. $\zeta_m(\mathbf{A})$ returns the eigenvector corresponding to the maximum eigenvalue of matrix $\mathbf{A}$ and $\mathbb{E}\{\cdot\}$ stands for the statistical expectation of a random variable. $\mathbf{I}_M$ denotes the $M \times M$ identity matrix.

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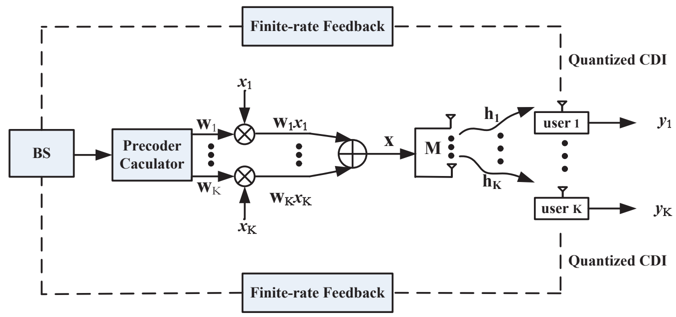

Fig. 1. System model of Multiuser MISO downlink with finite rate feedback.

II. System Model

We consider an MU-MISO system as shown in Fig. 1. Let $\mathbf{w}_i \in \mathbb{C}^{M\times 1}$ and $x_i \in \mathbb{C}$ be the precoder vector and data for user $i, i = 1, 2, \cdots, K$ respectively, then the received data at the kth user can be expressed by

$$y_k = \mathbf{h}_k \mathbf{w}_k x_k + \mathbf{h}_k \sum_{i=1, i\neq k}^{K} \mathbf{w}_i x_i + n_k, \quad k = 1, \cdots, K \quad (1)$$

where $\mathbf{h}_k \in \mathbb{C}^{1\times M}$ denotes the channel vector from the BS to user k. The entries of $\mathbf{h}_k$ are independent and identically distributed (i.i.d) complex Gaussian variables with zero mean and unit variance. $n_k \in \mathbb{C}$ is the Gaussian noise at user k with zero mean and variance N_0. The energy of transmitted symbol is normalized for each user, i.e. $\mathbb{E}\left\{x_k x_k^\dagger\right\} = 1$, hence the transmitted power is constrained by $\mathbb{E}\left\{\mathbf{w}_k^\dagger \mathbf{w}_k\right\} = P_k, k = 1, 2, \cdots, K$.

As in many literature like [4], [6]–[8], we assume that full information of $\mathbf{h}_k$ is obtained at the corresponding user k. Given $\mathbf{h}_k$, each user quantizes its normalized CSI $\tilde{\mathbf{h}}_k$, i.e. channel direction information (CDI) $\tilde{\mathbf{h}}_k = \mathbf{h_k}/\|\mathbf{h}_k\|$, into a predetermined codebook $\mathbf{C}_k \in \mathbb{C}^{M\times 2^B}$, which is also known at the transmitter. Here, B is the number of feedback bits. Afterwards, each user feeds back the index of the quantized version of CDI to the transmitter through a reversed link. After receiving the indices from all users, the BS will then recover the CSI by using the codebook. It is important to note that the obtained CSI at the BS is a delayed and quantized version of the real channel and the precoder vector for each user is calculated based on this obtained CSI.

III. Synthesized Channel Model

Before formulating the beamforming optimization problem in our system, we need to first introduce the channel model which contains the impacts of both CSI quantization error and feedback delay.

Without loss of generality, we consider the channel information between the BS and user k. According to the above-mentioned transmission procedure, user k has its CDI $\tilde{\mathbf{h}}_k[n-1]$ at time slot $(n-1)$ where $\tilde{\mathbf{h}}_k[n-1] = \mathbf{h}_k[n-1]/\|\mathbf{h}_k[n-1]\|$, and then quantizes its CDI to $\hat{\mathbf{h}}_k[n-1]$ by using the given codebook with B bits. After sending the index of its quantized CDI to the BS through a delayed feedback link, the BS obtains $\hat{\mathbf{h}}_k[n-1]$ and then uses this information for calculating multiuser beamforming vectors. As in [4], the relationship between the full CDI, i.e. $\tilde{\mathbf{h}}_k[n-1]$, and the quantized CDI, i.e. $\hat{\mathbf{h}}_k[n-1]$, can be expressed by

$$\tilde{\mathbf{h}}_k[n-1] = \sqrt{1 - z_k[n-1]}\hat{\mathbf{h}}_k[n-1] + \sqrt{z_k[n-1]}\mathbf{s}_k[n-1] \quad (2)$$

where $z_k[n-1] = 1 - \|\tilde{\mathbf{h}}_k[n-1]\hat{\mathbf{h}}_k^\dagger[n-1]\|^2$ and is independent of $\mathbf{s}_k[n-1] \in \mathbb{C}^{1\times M}$ which represents the quantization error vector with zero mean and unit norm isotropically distributed in the null space of $\hat{\mathbf{h}}_k[n-1]$. The expectation of $z_k[n-1]$ is given by $\mathbb{E}\{z_k[n-1]\} = \frac{M-1}{M}\delta$ with $\delta = 2^{-B/(M-1)}$.

Similar to [12], we also consider a stationary ergodic Gauss-Markov block fading channel where the CSI stays quasi-static during one symbol but changes independently between consecutive transmissions. By assuming only one symbol duration of the feedback link, i.e. the BS obtains $\hat{\mathbf{h}}_k[n-1]$ at time slot n after the user k sending it at time slot $(n-1)$, the relationship between the real channel at current time-slot n and the obtained CDI by the BS can be formulated by

$$\mathbf{h}_k[n] = \rho_k \|\mathbf{h}_k[n-1]\|\tilde{\mathbf{h}}_k[n-1] + \mathbf{e}_k[n] \quad (3)$$

where the obtained CDI by the BS, i.e. $\hat{\mathbf{h}}_k[n-1]$, is included in $\tilde{\mathbf{h}}_k[n-1]$ which has been given in (2). Hence, By substituting (2) into (3), we get the final synthesized channel model as follows

$$\begin{aligned}\mathbf{h}_k[n] =& \rho_k\sqrt{1 - z_k[n-1]}\|\mathbf{h}_k[n-1]\|\hat{\mathbf{h}}_k[n-1] \\ &+ \rho_k\sqrt{z_k[n-1]}\|\mathbf{h}_k[n-1]\|\mathbf{s}_k[n-1] + \mathbf{e}_k[n]\end{aligned} \quad (4)$$

which illustrated the relationship between the real channel $\mathbf{h}_k[n]$ in the current time-slot n and the available quantized CDI $\hat{\mathbf{h}}_k[n-1]$ at the BS. In (4), $\mathbf{h}_k[n]$ denotes the full CSI of current time-slot n and $\mathbf{e}_k[n]$, with i.i.d. entries $e_{ki}[n] \sim \mathcal{CN}\left(0, 1-\rho_k^2\right), i = 1, 2, \cdots, M$, is the channel error vector independent of $\mathbf{h}_k[n-1]$. According to Clarke's model, the correlation efficiency ρ_k is defined as $\rho_k = J_0(2\pi f_{dk} T_s)$, where $J_0(\cdot)$ is the zero-th order Bessel function of the first kind, f_{dk} denotes Doppler spread of user k and T_s is the length of the transmitted symbol.

IV. Precoding Based on Maximizing the Average SLNR

This section focuses on optimizing the multiuser beamforming design based on the synthesized channel model presented in (4). From (1), the SLNR at the kth user can be expressed by

$$\mathrm{SLNR}_k = \frac{\mathbf{w}_k^\dagger \mathbf{h}_k^\dagger \mathbf{h}_k \mathbf{w}_k}{\mathbf{w}_k^\dagger \left(\sum_{i=1, i\neq k}^{K} \mathbf{h}_i^\dagger \mathbf{h}_i\right)\mathbf{w}_k + N_0}. \quad (5)$$

From (5), the BS is unable to calculate the exact SLNR at current time slot n, denoted as $\mathrm{SLNR}_k[n]$, due to the absence of perfect CSI $\mathbf{h}_k[n]$, hence the beamforming vectors can not be optimized by directly maximizing SLNR as in conventional schemes in [9] and [10]. Therefore, we design the beamforming optimization problem by utilizing a modified SLNR-based object function instead of the primal SLNR expression in (5).

Based on the finite-rate feedback mechanism in our system, it is reasonable to consider the beamforming design via maximizing an expected SLNR averaged over the channel imperfection caused by channel quantization error and feedback delay. Mathematically, the beamforming optimization problem for user k can be formulated by

$$\begin{aligned}\max_{\mathbf{w}_k}\quad & \mathbb{E}_{\mathbf{h}_k[n]|\hat{\mathbf{h}}_k[n-1]}\{\mathrm{SLNR}_k[n]\}\\ \mathrm{s.t.}\quad & \mathbf{w}_k^\dagger\mathbf{w}_k = P_k.\end{aligned}\tag{6}$$

By substituting (4) and (5) into (6), we find it still very difficult to derive a closed-form expression of the objective function and thus hard to obtain the optimized beamforming vector with low-complexity algorithm. In order to tackle this problem, we resort to maximizing a lower bound to the expected SLNR in (6). By plugging (3) into (5) and utilizing *Jensen's inequality*, the expectation of (5) in (6) can be lower bounded by [1]

$$\begin{aligned}&\mathbb{E}_{\mathbf{h}_k[n]|\hat{\mathbf{h}}_k[n-1]}\{\mathrm{SLNR}_k[n]\}\\ =&\mathbb{E}\left\{\frac{\mathbf{w}_k^\dagger\mathbf{h}_k^\dagger[n]\,\mathbf{h}_k[n]\,\mathbf{w}_k}{\mathbf{w}_k^\dagger\left(\sum_{i=1,i\neq k}^K\mathbf{h}_i^\dagger[n]\,\mathbf{h}_i[n]\right)\mathbf{w}_k+N_0}\right\}\\ \geq&\frac{\mathbf{w}_k^\dagger\mathbb{E}\left\{\mathbf{h}_k^\dagger[n]\,\mathbf{h}_k[n]\right\}\mathbf{w}_k}{\mathbf{w}_k^\dagger\left(\sum_{i=1,i\neq k}^K\mathbb{E}\left\{\mathbf{h}_i^\dagger[n]\,\mathbf{h}_i[n]\right\}\right)\mathbf{w}_k+N_0}.\end{aligned}\tag{7}$$

Regarding (7) as the relaxation of $\mathbb{E}_{\mathbf{h}_k[n]|\hat{\mathbf{h}}_k[n-1]}\{\mathrm{SLNR}_k[n]\}$, we simplify the primal problem in (6) into the following optimization problem to design precoder $\mathbf{w}_k$

$$\begin{aligned}\max_{\mathbf{w}_k}\quad & \frac{\mathbf{w}_k^\dagger\mathbb{E}\left\{\mathbf{h}_k^\dagger[n]\,\mathbf{h}_k[n]\right\}\mathbf{w}_k}{\mathbf{w}_k^\dagger\left(\sum_{i=1,i\neq k}^K\mathbb{E}\left\{\mathbf{h}_i^\dagger[n]\,\mathbf{h}_i[n]\right\}\right)\mathbf{w}_k+N_0}\\ \mathrm{s.t.}\quad & \mathbf{w}_k^\dagger\mathbf{w}_k = P_k.\end{aligned}\tag{8}$$

To simplify the above problem, we need to first calculate the statistical expectations in (8). By first substituting (3) into $\mathbf{h}_k^\dagger[n]\,\mathbf{h}_k[n]$, we get

$$\begin{aligned}&\mathbb{E}\left\{\mathbf{h}_k^\dagger[n]\,\mathbf{h}_k[n]\right\}\\ =&\mathbb{E}\left\{\rho_k^2\|\mathbf{h}_k[n-1]\|^2\tilde{\mathbf{h}}_k^\dagger[n-1]\,\tilde{\mathbf{h}}_k[n-1]\right\}\\ &+\mathbb{E}\left\{\mathbf{e}_k^\dagger[n]\,\mathbf{e}_k[n]\right\}\\ &+\mathbb{E}\bigg\{\rho_k\|\mathbf{h}_k[n-1]\|\\ &\times\left(\tilde{\mathbf{h}}_k^\dagger[n-1]\,\mathbf{e}_k[n]+\mathbf{e}_k^\dagger[n]\,\tilde{\mathbf{h}}_k[n-1]\right)\bigg\}.\end{aligned}\tag{9}$$

Then, we calculate the three expectation terms in the above expression one by one. For notational simplicity, we drop out the time-slot indices and assume $\rho_k=\rho, k=1,2,\cdots,K$, i.e. each user has the same normalized Doppler spread $f_{dk}T_s$ according to the definition of ρ_k. Hence, the first term can be re-written as:

$$\begin{aligned}&\mathbb{E}\left\{\rho_k^2\|\mathbf{h}_k[n-1]\|^2\tilde{\mathbf{h}}_k^\dagger[n-1]\,\tilde{\mathbf{h}}_k[n-1]\right\}\\ =&\rho^2\mathbb{E}\left\{\|\mathbf{h}_k\|^2\tilde{\mathbf{h}}_k^\dagger\tilde{\mathbf{h}}_k\right\}\\ \overset{(a)}{=}&\rho^2\mathbb{E}\left\{\|\mathbf{h}_k\|^2\right\}\bigg(\mathbb{E}\{1-z_k\}\,\hat{\mathbf{h}}_k^\dagger\hat{\mathbf{h}}_k+\mathbb{E}\left\{z_k\mathbf{s}_k^\dagger\mathbf{s}_k\right\}\\ &\qquad+\mathbb{E}\left\{\sqrt{1-z_k}\sqrt{z_k}\left(\hat{\mathbf{h}}_k^\dagger\mathbf{s}_k+\mathbf{s}_k^\dagger\hat{\mathbf{h}}_k\right)\right\}\bigg)\\ \overset{(b)}{=}&\rho^2M\bigg(\mathbb{E}\{1-z_k\}\,\hat{\mathbf{h}}_k^\dagger\hat{\mathbf{h}}_k+\mathbb{E}\{z_k\}\,\mathbb{E}\left\{\mathbf{s}_k^\dagger\mathbf{s}_k\right\}\\ &\qquad+\mathbb{E}\left\{\sqrt{1-z_k}\sqrt{z_k}\right\}\left(\hat{\mathbf{h}}_k^\dagger\mathbb{E}\{\mathbf{s}_k\}+\mathbb{E}\left\{\mathbf{s}_k^\dagger\right\}\hat{\mathbf{h}}_k\right)\bigg)\\ \overset{(c)}{=}&\rho^2M\bigg(\left(1-\frac{M-1}{M}\delta\right)\hat{\mathbf{h}}_k^\dagger\hat{\mathbf{h}}_k\\ &\qquad+\frac{M-1}{M}\delta\left[\frac{1}{M-1}\left(\mathbf{I}_M-\hat{\mathbf{h}}_k^\dagger\hat{\mathbf{h}}_k\right)\right]\bigg)\\ =&\rho^2\left(M(1-\delta)\,\hat{\mathbf{h}}_k^\dagger\hat{\mathbf{h}}_k+\delta\mathbf{I}_M\right)\end{aligned}\tag{10}$$

where (a) holds because $\|\mathbf{h}_k\|$ is independent of $\tilde{\mathbf{h}}_k$, (b) holds because $\mathbb{E}\left\{\|\mathbf{h}_k\|^2\right\}=M$ and z_k is independent of $\mathbf{s}_k$, (c) holds because $\mathbb{E}\{z_k\}=\frac{M-1}{M}\delta$, $\mathbb{E}\{\mathbf{s}_k\}=\mathbf{0}$, and $\mathbb{E}\left\{\mathbf{s}_k^\dagger\mathbf{s}_k\right\}=\frac{1}{M-1}\left(\mathbf{I}_M-\hat{\mathbf{h}}_k^\dagger\hat{\mathbf{h}}_k\right)$ which has been proved in [6].

Concerning the second expectation term in (9), it is not hard to get:

$$\mathbb{E}\left\{\mathbf{e}_k^\dagger[n]\,\mathbf{e}_k[n]\right\}=\left(1-\rho^2\right)\mathbf{I}_M.\tag{11}$$

Then, by considering that $\mathbf{e}_k[n]$ is independent of $\mathbf{h}_k[n-1]=\|\mathbf{h}_k[n-1]\|\tilde{\mathbf{h}}_k[n-1]$ and $\mathbb{E}\{\mathbf{e}_k[n]\}=\mathbf{0}$, we can easily find that the expectation of the last item in (9) is zero.

At this step, we finally obtain the expectation of (9) as:

$$\begin{aligned}&\mathbb{E}\left\{\mathbf{h}_k^\dagger[n]\,\mathbf{h}_k[n]\right\}\\ &=\rho^2\left(M(1-\delta)\,\hat{\mathbf{h}}_k^\dagger\hat{\mathbf{h}}_k+\delta\mathbf{I}_M\right)+\left(1-\rho^2\right)\mathbf{I}_M\\ &=\rho^2M(1-\delta)\,\hat{\mathbf{h}}_k^\dagger\hat{\mathbf{h}}_k+\left[1-(1-\delta)\,\rho^2\right]\mathbf{I}_M.\end{aligned}\tag{12}$$

[1]For brevity, we drop the subscription $\mathbf{h}_k[n]|\hat{\mathbf{h}}_k[n-1]$ of $\mathbb{E}$ in the following of this paper.

With the above-derived result in (12) and denoting $C_1 = \rho^2 M(1-\delta), C_2 = 1-(1-\delta)\rho^2$, the optimization problem in (8) is equivalent to

$$\max_{\mathbf{w}_k} \frac{\mathbf{w}_k^\dagger \left(C_1 \hat{\mathbf{h}}_k^\dagger \hat{\mathbf{h}}_k + C_2 \mathbf{I}_M\right) \mathbf{w}_k}{\mathbf{w}_k^\dagger \left(C_1 \sum_{i=1, i\neq k}^{K} \hat{\mathbf{h}}_i^\dagger \hat{\mathbf{h}}_i + (K-1) C_2 \mathbf{I}_M\right) \mathbf{w}_k + N_0}$$
$$\text{s.t.} \quad \mathbf{w}_k^\dagger \mathbf{w}_k = P_k. \tag{13}$$

The above problem is shown as a generalized Rayleigh quotient problem [13]. Thus, the optimal solution to (13) can be calculated according to a closed-form expression by

$$\mathbf{w}_k = \sqrt{P_k} \times \zeta_m(\mathbf{\Gamma}_k) \tag{14}$$

where $\mathbf{\Gamma}_k = \left(C_1 \sum_{i=1, i\neq k}^{K} \hat{\mathbf{h}}_i^\dagger \hat{\mathbf{h}}_i + C_3 \mathbf{I}_M\right)^{-1} \left(C_1 \hat{\mathbf{h}}_k^\dagger \hat{\mathbf{h}}_k + C_2 \mathbf{I}_M\right)$ and $C_3 = (K-1) C_2 + N_0/P_k$.

Comparing (14) with the conventional scheme based on SLNR maximization given in [9] which requires perfect CSIT, we find that the computation difference between them lies in computing matrix $\mathbf{\Gamma}_k$ which is modified by some constant coefficients C_1, C_2 and C_3. As a result, we can conclude that the newly proposed scheme has the similar computational complexity with the conventional one in [9] while the proposed scheme requires only quantized CDI of users and is robust to both quantization error and feedback delay in finite-rate feedback systems.

V. Numerical Results

In this section, we will present several simulation results to justify the robustness of the proposed precoding scheme over the previous technique [9] and ZFBF in finite-rate feedback system consisting both quantization error and feedback delay. For the convenience of discussion, we denote our proposed precoding scheme based on maximizing the average SLNR as ASLNR and the scheme in [9] as SLNR. All the following simulations are based on $M = K = 4$ and randomly generated codebooks as in [4].

First, in order to individually investigate the impact of quantization error in finite-rate feedback to the performance of the system, we make a reduction to (14) by assuming $\rho = 1$, or equivalently $C_1 = M(1-\delta)$, $C_2 = \delta$, $C_3 = (K-1)\delta + N_0/P_k$, i.e. a reversed feedback link without delay. Under this condition, we get Fig. 2 which consists two cases, $B = 8$bits and $B = 16$bits. Since ZFBF scheme only perfectly eliminates the CCI and ignores the effect of noise while the schemes based on SLNR, including [9] and our proposed scheme, consider both CCI and noise, we can find from Fig. 2 that the performances of SLNR and ASLNR are much better than that of ZFBF. Moreover, since both ZFBF and [9] ignore the impact of quantization error in finite-rate feedback system when designing precoders, our proposed ASLNR scheme outperforms both ZFBF and SLNR, especially at high SNRs and a small B when the quantization error becomes the determinant of the system performance.

Next, let's add the impact of feedback delay into our discussion. To simplify the investigation and observe the impact

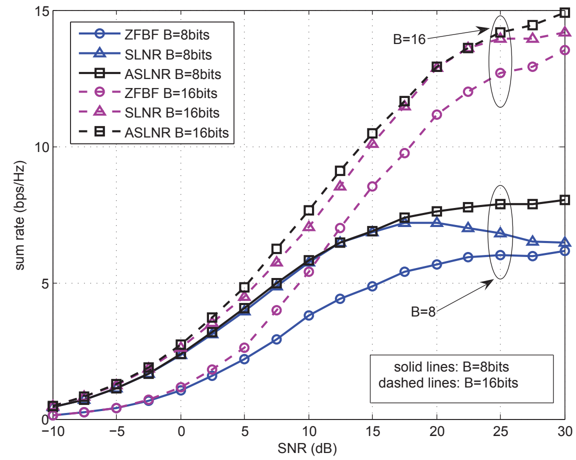

Fig. 2. Sum rate v.s. SNR under $M = K = 4$ without feedback delay.

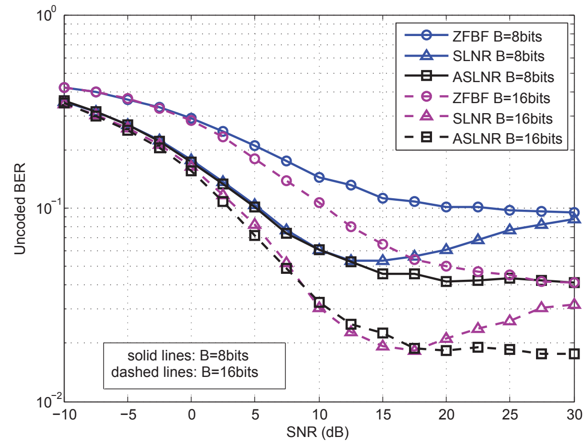

Fig. 3. Uncoder BER v.s. SNR under $M = K = 4$ and $fT = 0.05$.

of delay more clearly, we set the normalized Doppler spread of each user to be the same that $f_{dk}T_s = fT = 0.05, k = 1, 2, \cdots, K$, which means a feedback delay of about 1msec in a scenario with 50Hz Doppler spread, for instance. This time, we compare the uncoded BER of different precoding schemes in Fig. 3. Under the same impact of feedback delay, we can still find from Fig. 3 that the smaller the number of feedback bits, the more significant performance gain of ASLNR will have over the other two schemes, especially at high SNRs.

In Fig. 4 and Fig. 5, we place the different schemes into the case of the same number of feedback bits but different normalized Doppler spread to investigate the impact of delay. Fig. 4 with $B = 8$bits while Fig. 5 with $B = 16$bits. Take Fig. 5 as an example, the sum rate of ASLNR is increased over that of SLNR by over 20 percents in the case of $fT = 0.1$, while in Fig. 2 which assumes free delay, the increase is only about 5 percents. From this observation, we can conclude that when feedback delay is considered, our proposed scheme shows more effectiveness over [9]. If we compare Fig. 4 and Fig. 5, we can still find another two phenomena:

1) If we compare the corresponding solid lines (or dashed

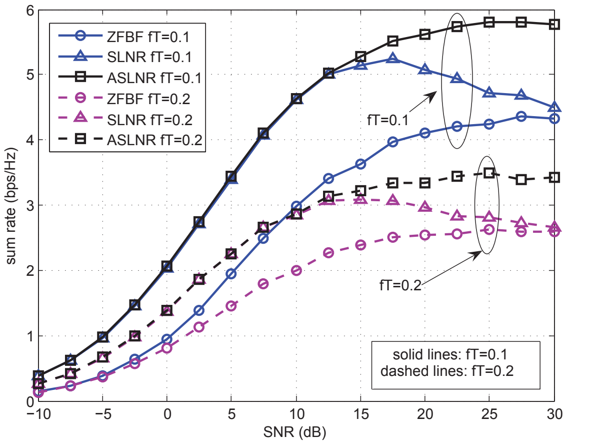

Fig. 4. Sum rate v.s. SNR under $M = K = 4$ and $B = 8$bits.

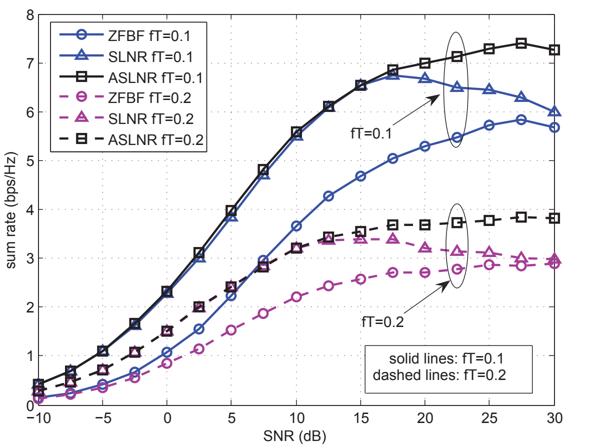

Fig. 5. Sum rate v.s. SNR under $M = K = 4$ and $B = 16$bits.

lines) in these two figures, a similar conclusion can be gathered with that we have mentioned in the discussion of Fig. 2 and Fig. 3 that the fewer the feedback bits, the larger advantage our proposed scheme will have over the schemes of SLNR and ZFBF.

2) The performance of the system is more sensible to the feedback delay than to quantization error since the number of feedback bits in Fig. 5 doubles that in Fig. 4, while the increase of sum rate in Fig. 5 over that in Fig. 4 is not that many as we may expect. This may guide us to make more investigations in the future to deal with the problem of feedback delay.

At last, another more interesting discovery we find is that, either the precoding scheme based on maximal SLNR proposed in [9] and [10]or the conventional MMSEBF scheme in [5], if we apply it directly into the practical finite-rate feedback system where only a delayed and quantized version of CSI is available at the BS, both of them will suffer a performance degradation after a certain SNR, such as approximately 15dB in Fig 2–5 (as for the case of MMSEBF, please refer to [6] or [8]). However, if we make a revise to the original criterion and take advantage of the statistical properties of the CSI, like the proposed scheme in this paper as well as those robust MMSEBF schemes investigated in [6]– [8], this performance degradation can be largely mitigated. This is also the main idea behind this paper.

VI. Conclusion

Due to the effect of quantization error and feedback delay, the precoding scheme presented in [9] or [10] will largely suffer a performance degradation if we apply it directly into the limited feedback system. In this paper, we revised the criterion used in [9], [10] and proposed a new one, i.e. maximizing the average SLNR to generate a new precoder expression. Mathematical analysis attested that the proposed precoding scheme has similar computational burdens as the previous one, while numerical results justified that the robustness of our proposed scheme outperforms the previous one as well as ZFBF in the application of finite-rate feedback systems.

VII. Acknowledgement

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